SYNOPSIS

Title : "A STUDY OF SOME PROBLEMS ON ALGEBRAIC GROUP AND ITS IMPACT ON RING THEORY"

(a) Introdution

Modern algebra is a fundamental area of mathematics devoted to the study of algebraic structures such as groups, rings, fields, modules, vector spaces, and algebras. Rather than concentrating on numerical calculations, we focuses on structural properties and the relationships among these systems. This field supports a wide range of mathematical disciplines including number theory, topology, geometry, representation theory and plays a crucial role in areas such as theoretical computer science, cryptography and mathematical physics etc.

Algebraic group is the analogue in modern algebra of ring theory or a Lie group in case of Noncommutative rings and division algebra respectively. It provides a unifying language for some significant results in modern algebra and Ring theory. We propose to extend ring theory in algebra and number theory which generalizes Galois cohomology. Our main purpose is to discuss the properties of artinian ring, Noetherian and semiprime ring of extensions of a connected derivations and automorphism group. We focus is on derivation of modified version of Galois cohomology in the context of Lie algebra. We propose to extend the connections between Galois theory and areas of geometry and topology, including Grothendieck's version of Galois theory derive by T. Szamuely. Ring theory is a major area of modern algebra that studies rings-algebraic structures equipped with addition and multiplication analogous to those on the integers. We extend the internal structure of rings, their associated modules, and specialized classes such as group rings and division rings. Particular emphasis is placed on commutative rings, which play a vital role in applications across algebraic geometry and algebraic number theory.

(b) Work Plan

We propose to extend the non-commutative structures by utilizing the close relationship between Hopf algebras and operator algebras. Our

approach seeks to extend identified equivalence results between C*-algebras and von Neumann algebras, and to derive C*-algebraic formulations for certain locally compact algebras and non-commutative von Neumann algebras. Runde and M. Cowling introduce the modern algebras are generally not contractible, while ring theory presents additional challenges when dealing with operator algebras arising from module theory.

We extend the result of Galois theory builds on this by studying field extensions using the group of automorphisms of the extension to understand the symmetries of polynomial equations. We propose to derive the properties include the relationship between subgroups of the Galois theory and intermediate fields theory, which is formalized by the fundamental theorem of Galois Theory derive by Artin and H.M. Edwards. The set of all automorphisms of a field extension forms a group under composition is the Galois group, $Gal(L/K)$. An automorphism is a bijective homomorphism that fixes the base field K.

We propose to extend the study of module categories and homological dimensions by building on fundamental group-theoretic concepts. Our goal is to broaden the algebraic framework of Morita equivalence and further develop its implications within ring theory. We aim to generalize related operations and theorems, establishing significant results on ring decompositions. Furthermore, we seek to extend the homological algebra of the integers to a wider class of algebraic systems with a particular focus on rings.

(c) Details of the project with chapterisation

We propose to generalize the concept of ring homomorphisms theory by building on an analogy with quotient groups formed by normal subgroups, thereby extending ring theory in a similar structural direction. Additionally, we note that the moduli space of complex algebraic curves occupies a central position in modern mathematics. We introduce the concepts of reduction order and reduction-factorization on words and use them to show that any right or left Noetherian pointed Hopf algebra over k is affine. This provides a partial affirmative solution to the classical affineness problem for Noetherian Hopf algebras posed by Wu

Chapter - 4 Properties of Homological Algebra and category theory

Homological algebra is a field of mathematics that using chain complexes, exact sequences, and derived functors. It provides powerful tools for analyzing modules, rings, and other algebraic systems by measuring and encoding their structural properties through homology and cohomology. Category theory derive the conceptual foundation - through notions such as functors, natural transformations, limits and adjoint pairs while homological algebra applies these concepts representation theory, algebraic geometry, and beyond.

(d) Hypothesis

We prpopose to derive affine Noetherian PI Hopf algebra is Artin–Schelter Gorenstein, and formulate a classical affineness question regarding the structure of Noetherian Hopf algebras by Wu and Zhang. The importance of infinite-dimensional Hopf algebras that satisfy natural finiteness conditions, such as being affine and Noetherian, having finite Gelfand–Kirillov dimension, being Artin–Schelter regular, or possessing the Calabi–Yau property. Another area of focus is the study of the group of units in a ring and the corresponding definition of a field. We derive the unit groups of rings that arise in modular representation theory and subsequently examine zerc-divisors and integral domains. we formulate non-commutative structures with those of classical frameworks, including conformal mappings, the modular group, module categories, and homological dimensions.

(e) Motivation

The proposed research project is entirely theoretical in nature. We have access to the necessary reference materials in the Central Library of B.N.M. University, along with additional contemporary and classical books and journals. We may also consult resources available in the Science Section of the National Library in Kolkata and at the Indian Statistical Institute.

and Zhang. For a pointed Hopf algebra Hover k, further we explain a well-ordered set X such that:(1) H is generated, as an algebra, by the subset X_l of irreducible letters, and (2) X_l is finite whenever H is right Noetherian.

The proposed research project is organized into four different chapters, each chapter highlighting its wide-ranging applications across various allied areas of mathematics. The chapters are as follows:

Chapter - 1 Properties of Non-commutative Rings and division algebra

These rings arise naturally in many areas of algebra, geometry, and mathematical physics and they often exhibit richer and more complex behavior than their commutative counterparts. A division algebra is a specific types of non-commutative ring in which every nonzero element possesses a multiplicative inverse, thereby extending the concept of a field without imposing commutativity on multiplication.

Chapter - 2 Properties of Galois cohomology and its impact on Lie algebra

Galois cohomology serves as a powerful tool for analyzing the action of a Galois group on algebraic structures, uncovering hidden symmetries and identifying obstructions within field extensions. When applied to Lie algebras, it becomes essential for understanding their behavior over different fields, especially those that are not algebraically closed. It aids in classifying various forms of Lie algebras, studying their automorphism groups, and describing central extensions and deformation phenomena. In this way, Galois cohomology offers profound insights into the structural, classificatory, and arithmetic aspects of Lie algebras.

Chapter - 3 Structure of Ring theory and its applications

Ring theory is a branch of abstract algebra focuses on the internal structure of rings, ideals, homomorphisms, and modules, as well as special classes such as commutative rings, division rings, and group rings. Ring theory forms the foundation for many areas of mathematics, including algebraic geometry, number theory, functional analysis, and representation theory.

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